Approximation of the number of roots that do not lie on the unit circle of a self-reciprocal polynomial

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Abstract. We introduce the ratio of the number of roots not equal to 1 in modulus of a reciprocal polynomial $R_d(x)$ to its degree d. For some sequences of reciprocal polynomials we show that the ratio has a limit L when d tends to infinity. Each of these sequences is defined using a two variable polynomial $P(x, y)$ so that $R_d(x) = P(x, x^n)$. For $P(x, y)$ we present the theorem for the limit ratio which is analogous to the Boyd-Lawton limit formula for Mahler measure. We present a double integral formula for approximation the limit ratio. In a previous paper we have calculated the exact value of the limit ratio of polynomials correlated to many bivariate polynomials having small Mahler measure introduced by Boyd and Mossinghoff. We demonstrate here that the double integral formula gives the value very close to the exact value (the error is $< 10^{-5}$. We show that the limit ratio of the sequence $P(x, x^n)$ is not always equal to the limit ratio of the sequence $P(y^n, y)$ unlike Mahler measure.

Keywords: Reciprocal polynomial; envelope; unimodular roots.

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